

ASSIGNMENT 6

Reading:

105 Notes 7.1-7.8

Hand & Finch 4.1-4.6

1.

Show that the period of oscillation of a particle of mass m in a potential $U = A|x|^n$ is given by

$$T = \frac{2}{n} \sqrt{\frac{2\pi m}{E}} \left(\frac{E}{A}\right)^{1/n} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{2} + \frac{1}{n})}$$

Take $n = 2$, evaluate the gamma functions, and thus show that T reduces to the normal expression for a parabolic potential.

2.

Use a Green function to obtain the response of an underdamped linear oscillator

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = F(t)$$

to a driving (acceleration) function of the form

$$F(t) = 0 \ (t < 0); = F_0 e^{-\beta t} \ (t > 0) ,$$

where γ , ω_0 , F_0 , and β are constants.

3.

Consider a nonlinear damped oscillator whose motion is described by

$$\frac{d^2 x}{dt^2} + \lambda \frac{dx}{dt} \left| \frac{dx}{dt} \right| + \omega_0^2 x = 0$$

The initial conditions are $x(0) = a$, $\dot{x}(0) = 0$. Use the method of perturbations to find a solution that is accurate to first order in the small quantity λ .

4.

Two particles moving under the influence of their mutual gravitational force describe circular orbits about one another with period τ . If they are suddenly stopped in their orbits and allowed to gravitate toward each other, show that they will collide after a time $\tau/(4\sqrt{2})$.

5.

A spacecraft in uniform circular orbit about the sun, far from any planet, consists of a nose cone and a service module. By means of explosive bolts, the nose cone separates from the service module. The direction of motion of the nose cone is unchanged, but its orbit becomes a parabola instead of a circle. The service module falls directly into the sun. Solve for the ratio $\rho = m_{\text{cone}}/m_{\text{spacecraft}}$, where the spacecraft mass is considered to be the sum of the nose cone and service module masses.

6.

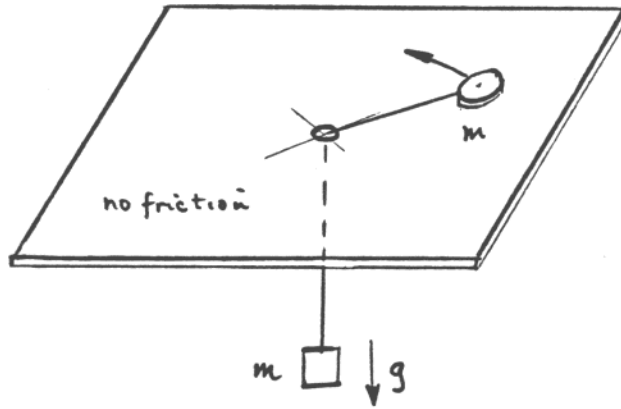
A particle moves under the influence of a central force given by $F(r) = -k/r^n$. If the particle's orbit is circular and passes through the force center, show that $n = 5$.

7.

A spacecraft in circular orbit about the sun fires its thruster in order to change instantaneously the *direction* of its velocity \mathbf{v} by 45° (toward the sun), keeping the same *magnitude* $|\mathbf{v}|$. What is the eccentricity of the spacecraft's new orbit?

8.

A puck of mass m is connected by a massless string to a weight of the same mass. It moves without friction on a horizontal table, in circular orbit about the hole.



(a)

Calculate the frequency of small radial oscillations about the circular orbit.

(b)

Expressing this frequency as a ratio to the orbital frequency, show that the orbit does not close.